Real Options

Olivier Levyne (2020)

DCF limits and usefulness of Real Options

Limits of the DCF approach

- Possibility to fine-tune the discount rate i.e. the WACC according to the assumptions that are taken into account for the market risk premium and for the beta
- Uncertainty of future FCF
- Book value of debt versus economic value of equity

Usefulness of Real Options for Corporate Valuation purpose

- In options pricing models (Black & Scholes, Cox-Ross-Rubinstein…)
  - Discounting based on an undisputable risk-free rate
  - No use to estimate future FCF: only their volatility is considered
- Possibility to get the economic value of debt based on an option pricing models

Other applications for valuation purpose: option to exit, patent, option du exit a joint venture, oil field concession…

Equity value according to Black & Scholes


S = EV	120
E = D	100
r discrete	2,00%
r continuous	1,98%
τ	10
σ	40%
d_1	0,93
d_2	-0,33
$\Phi(d_1)$	0,82
$\Phi(d_1)$	0,37
Probability of bankruptcy	63%
C = Equity by B&S	**69**

- Assumption: debt = zero coupon
- Implicit right for the shareholders
  - Repay the debt to buy the assets, when the debt is maturing, if the EV is higher than the nominal value of the debt to be repaid (D)
  - Abandon the firm to its lenders, if EV < D, thanks to the limited liability of shareholders
- Consequence: wealth of shareholders = premium of a call on assets, its strike price being the nominal value of the debt to be repaid
  - S = spot price of the underlying asset = EV
  - E = strike price = amount to be paid should the call be exercised = D
  - τ = debt's maturity, in years
  - σ = volatility of the underlying asset = EV's volatility
  - r = risk-free rate, in continuous time
- Formula : Equity value = $EV.\Phi(d_1) - De^{-r\tau}\Phi(d_2)$

$$d_1 = \frac{\ln\left(\frac{EV}{D}\right) + \left(r + \frac{\sigma^2}{2}\right).\tau}{\sigma\sqrt{\tau}}, d_2 = d_1 - \sigma\sqrt{\tau}$$

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

Nota: $\Phi(x)$ *is provided by Excel*: *normsdist*(x)

Debt value and Merton's contributions

EV	120
Debt (face value)	100
r continuous	2%
τ (time to expiration)	10
σ(A)	40%
$\Phi(d_1)$	0,83
$\Phi(d_2)$	0,37
Equity value	69
Probability of default	62,9%
Economic value of debt = EV - Equity value	**51,34**
Economic value of unrisky debt = PV of debt's face value (using r)	81,87
Recovery rate given default = $\Phi(-d_1)/\Phi(-d_2)$	**28%**
Recovery given default = $[\Phi(-d_1)/\Phi(-d_2)]$.EV	33,36
LGD = Economic value of unrisky debt - Recovery given default	**48,51**
Expected LGD = $\Phi(-d_2)$.LGD	30,53
Check: economic value of unrisky debt - expected LGD	51,34
$\Phi(-d_1)$	0,17
d=D.exp(-rt)/V	0,68
1/d	1,47
Spread	**4,7%**
Cost of debt all in	6,7%

- Notations
  - D = nominal value of the debt to be repaid
  - B = economic value of debt
- Reminder: Equity value = $EV.\Phi(d_1) - De^{-r\tau}\Phi(d_2)$
- $\Phi(d_2)$ = probability for the shareholders to exercise their call = probability for the firm to be "*in bonis*"
- 1- $\Phi(d_2) = \Phi(-d_2)$ = probability of bankruptcy
- B = EV – Equity value
- B = $EV.\Phi(-d_1) + De^{-r\tau}\Phi(d_2)$
- Spread on corporate debt = R (full cost of debt) - r (risk free rate)
- R – r = $-\frac{1}{\tau}\ln[\Phi(d_2) + \frac{EV}{De^{-r\tau}}\Phi(-d_1)]$
- Breakdown of the economic value of debt

$$B = De^{-r\tau} - \Phi(-d_2)\left[De^{-r\tau} - \frac{\Phi(-d_1)}{\Phi(-d_2)}EV\right]$$

$\frac{\Phi(-d_1)}{\Phi(-d_2)}$ = recovery rate given default

$De^{-r\tau} - \frac{\Phi(-d_1)}{\Phi(-d_2)}EV$ = Loss Given Default

Option to expand

- Acquisition of a subsidiary in Uruguay to test the South American market
  - Price consideration: 100
  - DCF valuation: 90
  - NPV = -10
- Investment in Uruguay to be looked upon as an option to buy a bigger subsidiary in 3 years in Brazil for a consideration of 1000 (to be paid in 3 years), whereas its DCF value, which has just been calculated, is 900. The volatility of its FCF is 40% and the risk-free rate is 2%
  - E = 1000
  - S = 900
  - τ = 3 years
  - σ = 40%
  - r = 2%
- Value based on Black & Scholes = 229
- Adjusted NAV = -10 + 229 = 119 > 0

S	900
E	1000
r discrete	2,00%
r continuous = ln(1+ r discrete)	1,98%
τ	3
σ	40%
d1	0,28
d2	-0,41
$\Phi(d_1)$	0,61
$\Phi(d_2)$	0,34
C by B&S	**229**

Patent's value

S = EV	800
Annual cost of delay = 1/τ = q	10%
S' = EV.exp$^{-1/\tau.\tau}$ = EV.e^{-1}	294
E = I_0	1000
r discrete	2,00%
r continuous	1,98%
τ	10
σ	40%
d1	-0,18
d2	-1,44
$\Phi(d_1)$	0,43
$\Phi(d_2)$	0,07
Expected future value of EV = EV.e^{rt}.$\Phi(d_1)$	154
Expected cash outfow = I_0.$\Phi(d_2)$	75
EV.e^{rt}.$\Phi(d_1)$-I_0.Φ(d2)	80
e^{-rt}.[EV.e^{rt}.$\Phi(d_1)$-I_0.Φ(d2)]	65
C = Value of the patent	**65**

- Assumptions
  - Possibility to buy a patent that will enable to manufacture a new drug
  - CAPEX to equip the factory that will manufacture the drug: 1000
  - Sum of present values of CF to be generated by the project: 800
  - Volatility of CF = 40%
  - Lifetime of the patent: 10 years
  - Risk free rate: 2%
- Patent to be looked upon as an option to equip the factory for a a consideration of 1000
  - Investments to be performed when the NPV (currently amounting to 800-1000=-200) will be positive
  - Possibility for the sum of present values of CF to increase and reach at least 1000, thanks to their volatility
  - Merton's formula to be used in order to include the annual cost of delay $\left(\frac{1}{\tau}\right)$, to be looked upon as a dividend yield (δ) from an option pricing model's point of view: replacement, in the Black and Scholes formula, of S by S' with

$$S' = Se^{-\delta\tau} = Se^{-\frac{1}{\tau}.\tau} = \frac{S}{e}$$

Value of an oil field concession

Option ref	1	2		1	2	3	4	5	6	7	8	9	10
S_0	93	93		93	93	93	93	93	93	93	93	93	93
Convenience yield q	0,00%	0,00%			0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
$S_0.e^{-qt}$	93	93			93	93	93	93	93	93	93	93	93
E	50	50		50	50	50	50	50	50	50	50	50	50
r discrete		2,00%			2,00%	2,00%	2,00%	2,00%	2,00%	2,00%	2,00%	2,00%	2,00%
r continuous		1,98%			1,98%	1,98%	1,98%	1,98%	1,98%	1,98%	1,98%	1,98%	1,98%
σ		80,0%			80%	80%	80%	80%	80%	80%	80%	80%	80%
τ		5			1	2	3	4	5	6	7	8	9
d_1		1,30			1,20	1,15	1,18	1,24	1,30	1,36	1,42	1,48	1,53
d_2		-0,49			0,40	0,02	-0,20	-0,36	-0,49	-0,60	-0,70	-0,79	-0,87
$\Phi(d_1)$		0,90			0,89	0,87	0,88	0,89	0,90	0,91	0,92	0,93	0,94
$\Phi(d_2)$		0,31			0,66	0,51	0,42	0,36	0,31	0,27	0,24	0,22	0,19
C per barrel in $	43	70		43	50	57	62	66	70	73	75	77	79
Output capacity	5	5		1	1	1	1	1	1	1	1	1	1
C in M$	215	349		43	50	57	62	66	70	73	75	77	79
Value of the concession (M$)		**564**			**653**								
Number of decisions to open the tap or not			1	2	10								
Value of the concession (M$)			430	564	653								

Increasing value of flexibility →

- RFP to get the concession of an oil field for 10 years
  - Spot price of 1 barrel: 93 $
  - Full cost to product 1 barrel: 50 $
  - Volatility of oil: 80%
  - Risk-free rate: 2%
  - Installed capacity: 1 million barrels per year
- Periodicity of the decision to open the tap or not
  - Once a year: then concession's value = value of a portfolio of 10 options to open the tap, the 1st one being immediately exercised or not
  - Every 5 years: then concession's value = value of a portfolio of 2 options to open the tap, the 1st one being immediately exercised or not
  - Once i.e. now: then concession's value = value of 1 call that has no time premium

    = (93 – 50) x 1 000 000 x 10 = 430 M$
  - Assumed no convenience yield